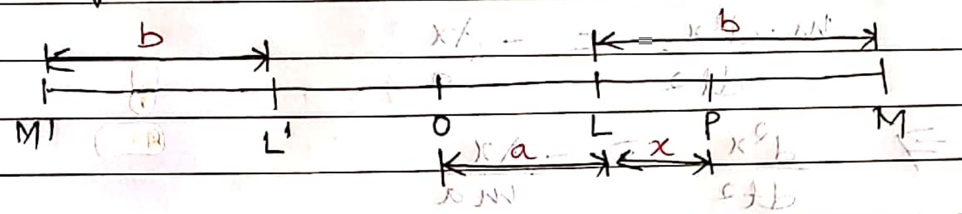
HORIZONTAL ELASTIC STRING

Theorem:

One end of an elastic string whose modulus of elasticity is λ and whose natural length is 'a' is tied to a fixed point on a smooth horizontal table and the other end is tied to a particle of mass 'm' lying on the table. The particle is pulled to a distance where the extension of the string becomes 'b' and then let go. To discuss the motion and to show that the period of one complete Osiullation is

$$2\left(\pi + \frac{2a}{b}\right)\sqrt{\frac{am}{\lambda}}.$$

Proof:-



Let $OL = a$ be the natural length of the string whose end O is fixed on a smooth horizontal table and on the other end L is tied the particle of mass 'm' is tied and is pulled to M such that $LM = b$ and then let go.

The particle will move towards O due to the tension of the string

Let P be the position of the particle at any time of its motion such that

$LP = x$

$\therefore$ The time from M to P is 't'

Let T be the tension of the string at P

Therefore, by Hooke's law

$$T = \lambda \frac{x}{a} \quad \text{--- (1)}$$

Clearly,
T is the only force acting on the particle towards O and therefore, the eq^n of motion

$$m \cdot \frac{d^2x}{dt^2} = -T$$

– ve sign is taken as x decreases with time

$$m \cdot \frac{d^2x}{dt^2} = -\frac{\lambda x}{a}$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{\lambda x}{ma}$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\mu x \quad \text{--- (2)}$$

where, $\mu = \frac{\lambda}{ma}$

Which shows that the motion of the particle is a S.H.M. about L with time period $\frac{2\pi}{\sqrt{\mu}}$

$$\frac{2\pi}{\sqrt{\mu}} = \frac{2\pi}{\sqrt{\frac{\lambda}{ma}}} = 2\pi\sqrt{\frac{ma}{\lambda}}$$

(2) $\Rightarrow \frac{d}{dt}\left(\frac{dx}{dt}\right) = -\mu x$

$\Rightarrow \frac{d v}{dt} = -\mu x$

$\Rightarrow \frac{dv}{dx} \times \frac{dx}{dt} = -\mu x$

$\Rightarrow v\,dv = -\mu x\,dx$

Integrating both sides, we get

$\int v\,dv = -\mu \int x\,dx$

$\Rightarrow \frac{v^2}{2} = -\frac{\mu x^2}{2} + C_1$

Initially at M,

$v = 0, \; x = b$

$\therefore \quad 0 = -\frac{\mu b^2}{2} + C_1$

$\Rightarrow \quad C_1 = \frac{\mu b^2}{2}$

$\therefore \quad \frac{v^2}{2} = -\frac{\mu x^2}{2} + \frac{\mu b^2}{2}$

$\Rightarrow v^2 = \mu(b^2 - x^2)$

$\therefore v = -\sqrt{\mu}\sqrt{b^2 - x^2}$ —— (3)

$-ve$ sign is taken because x decreases with time.

Now at L,

$x = 0$

(2) $\Rightarrow \frac{d^2x}{dt^2} = 0$

(3) $\Rightarrow v = -\sqrt{\mu}\sqrt{b^2}$

$= -\sqrt{\mu}\, b$

which shows that the particle will move to the left of L with uniform velocity $\sqrt{\mu}\, b$. reaches to L' such that

$LL' = 2a$

and the time taken by the particle from L to $L' = \frac{2a}{v}$

$= \frac{2a}{\sqrt{\mu}\, b}$

$= \frac{2a}{b}\sqrt{\frac{mg}{\lambda}}$

Now after L'

The string begins to extend and the tension comes into action and the motion of the particle is again a S.H.M. with amplitude $L'M' = LM = b$ and the time period,

$T = \frac{2a}{b}\sqrt{\frac{mg}{\lambda}}$

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Also, the time taken by the particle from L to L'

$$= 2 \times \frac{2a}{b}\sqrt{\frac{ma}{\lambda}}.$$

$\therefore$ The total time of one complete oscillation is

$$2\pi\sqrt{\frac{ma}{\lambda}} + 2\times\frac{2a}{b}\sqrt{\frac{ma}{\lambda}}.$$

$$= 2\sqrt{\frac{ma}{\lambda}}\left[\pi + \frac{2a}{b}\right]$$

$$= 2\left(\pi + \frac{2a}{b}\right)\sqrt{\frac{ma}{\lambda}}$$